Algebraic Relations among Matching Constraints of Multiple Images

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Abstract: Given a set of $n \geq 2$ uncalibrated views, for any corresponding point across n views, there exist three types of matching constraints: bilinear constraints (for $n \geq 2$), trilinear constraints (for $n \geq 3$, [12]) and quadrilinear constraints (for $n \geq 4$, [6, 14, 3]). The exact algebraic relations among these multi-linear constraints have not been elucidated by previous authors. This paper examines the relations between these matching constraints by singling out the degenerate view and point configurations. The key result that will be established is that for generic view configurations and generic points, all multi-linear constraints may algebraically be reduced to the algebraically independent bilinear constraints. In other words, all matching constraints are contained in the ideal generated only by the bilinear constraints for generic views and points. As a consequence, $2n-3$ algebraically independent bilinearities from pairs of views completely describe the algebraic/geometric structure of n uncalibrated views for generic views and points. For degenerate points of generic views, each type of constraint reduces differently. The exact reduced form of the matching constraints are also made explicit by computer algebra.

Key-words: geometry, invariant, epipolar geometry, bilinearity, trilinear ity, uncalibrated image.

(Résumé : tsvp)

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Relations algébriques entre les contraintes géométriques d'images multiples

Résumé : Il est connu qu'il existe 3 types de contraintes géométriques pour les points en correspondence dans les images multiples: relations bilinéaires, trilinéaires et quadrilinéaires. Nous démontrons que toutes les relations multilinéaires se re'duisent en relations bilinéaires pour les configurations générales des points et des caméras. Les formes réduites et exactes des relations trilinéaires sont aussi exhibées par le calcul symbolique pour les points en configurations dégénérées et les caméras en configurations générales.

Mots-clé : géométrie, géométrie épipolaire, bilinéarité, trilinéarité, caméra non-calibrée

1 Introduction

Recently, a number of works have been concentrated on the analysis of multiple uncalibrated images [2, 7, 10, 11]. Thanks to the pioneering work of Shashua [12], Hartley [6], Triggs [14], Faugeras and Mourrain [3, 4], Luong and Vieville [9, 15] and Werman and Shashua [16] on the study of the geometric relationships between different uncalibrated views, it has been shown that there are only three types of algebraic relations on image points between any uncalibrated views: Bilinearities for two views, trilinearities for three views and quadrilinearities for four or more. It has also been well established that quadrilinearities are not independent; they can be generated from (independent) trilinearities and bilinearities [14, 3] and further more bilinearities can be generated by trilinearities. However it is unclear if trilinearities can be generated by (independent) bilinearities. In [14], by geometric arguments, it is suggested that the trilinearities are generated by the bilinearities. In this paper, the exact relations among those multi-linear constraints will be studied. One key idea is to introduce the generic/degenerate view configurations and point configurations and make distinctions between these two different kinds of degenerate configurations. This allows us to establish that all trilinearities follow the bilinearities for generic point and view configurations. Therefore, it can be concluded that all multi-linear constraints may be reduced to the independent bilinearities. For degenerate points, all constraints are reduced, and we will give the exact reduced forms of the matching constraints.

This paper is organised as follows. Section 2 reviews the derivations of multi-linear constraints. Then the case of three views will be studied in detail in Section 3 in which some major results will also be presented. After that, Section 4 basically extends the resuls of three views to the general n view case. Finally, some concluding remarks are given in Section 5.

2 Derivation of multi-linear matching constraints: review

Following the general formalism proposed by Faugeras [3] and Triggs [14] for deriving the algebraic relations among views, the multi-linear constraints may be derived as follows.

For each view, consider a point $\mathbf{x}^T = (x, y, z, t)^T$ in $\mathcal{P}^3$ projected onto a point $\mathbf{u}^T = (u, v, w)^T$ in $\mathcal{P}^2$ by a 3×4 matrix $\mathbf{P}_{3\times4} = (p_{ij})$ as

$$\lambda(u, v, w)^T = P_{3\times4}(x, y, z, t)^T, \tag{1}$$

for short, $\mathbf{u}^T = P_{3\times4}\mathbf{x}^T$. Then equation (1) can be rewritten in terms of $\mathbf{x}$ as

$$\begin{pmatrix} u\mathbf{p}_3^T - w\mathbf{p}_1^T \\ v\mathbf{p}_3^T - w\mathbf{p}_2^T \end{pmatrix} \mathbf{x} = 0,$$

where $(\mathbf{p}_1^T\ \mathbf{p}_2^T\ \mathbf{p}_3^T)^T = P_{3\times4}$.

Now, taking $\mathbf{P}_1 = (a_{ij})$, $\mathbf{P}_2 = (b_{ij})$, $\mathbf{P}_3 = (c_{ij})$, ..., as projection matrices for n views, we obtain

$$G_{2n\times 4}\mathbf{x}^T = \begin{pmatrix} u_1\mathbf{a}_3^T - w_1\mathbf{a}_1^T \\ v_1\mathbf{a}_3^T - w_1\mathbf{a}_2^T \\ u_2\mathbf{b}_3^T - w_2\mathbf{b}_1^T \\ v_2\mathbf{b}_3^T - w_2\mathbf{b}_2^T \\ u_3\mathbf{c}_3^T - w_3\mathbf{c}_1^T \\ v_3\mathbf{c}_3^T - w_3\mathbf{c}_2^T \\ \cdots \end{pmatrix} \mathbf{x}^T = 0.$$

This corresponds to what Triggs called the reduced reconstruction equation. All matching constraints come from the fact that the matrix is of rank three. This is equivalent to saying that all 4×4 determinants of $G_{2n\times 4}$ are vanishing. Let $[ijkl]$ denote the 4 by 4 determinant of $i, j, , k, l$-th row vectors of $G_{2n\times 4}$. All these determinants may be divided into three classes each of which gives one type of constraint, for more details, *cf.* [14, 3].

For instance, in the case of three views, we have 3 bilinearities which involve only two views by taking two rows from the same view:

$$B = \{bb_1 = [1234], bb_2 = [1256], bb_3 = [3456]\}$$

and 12 trilinearities which involve three views:

$$T = T_1 \bigcup T_2 \bigcup T_3,$$

where

$$\begin{aligned} T_1 &= \{tt_1 = [1235], tt_2 = [1236], tt_3 = [1245], tt_4 = [1246]\}, \\ T_2 &= \{tt_5 = [3415], tt_6 = [3416], tt_7 = [3425], tt_8 = [3426]\}, \\ T_3 &= \{tt_9 = [5613], tt_{10} = [5614], tt_{11} = [5623], tt_{12} = [5624]\}. \end{aligned}$$

The 4 trilinearities of T_1 are initially proposed by Shashua [12], which are *linearly* independent. When more than 4 views are given, by taking each row from a different view, we obtain the quadrilinear constraints, for instance, one example may be given as $qq_1 = [1357]$. We denote all quadrilinearities by $Q = \{qq_1, \ldots\}$.

In the remaining of the paper, an ideal generated by a set of polynomials $S = \{s_1, s_2, \ldots, s_n\}$ is denoted as $\mathbf{I}_S =< s_1, s_2, \ldots, s_n >$.

Before giving some general results, we will first study the case of three views, therefore the relationship beween bilinearities and trilinearities, then with the same idea, we extend this to the general n view case.

3 Three views: Relation between bilinearities and trilinearities

All possible matching constraints of three views are generated by the bilinearities and trilinearities. Obviously, not all of them are algebraically independent. There exist quite complicated algebraic relations among the minors as explicated in [13]. It is known as the syzygy ideal if we consider the polynomial ring $\mathbb{R}[g_{ij}]$. Note that we are considering the constraints in the ring $\mathbb{R}[a_{ij}, b_{ij}, c_{ij}, \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$ which is slightly different.

It is widely accepted [12, 14, 3] that the bilinearities can be generally generated by the trilinearities and it is expected that the trilinearities might be *generally* generated by the bilinearities. However the exact relationship between them remains unclear. It is the main purpose of this section to clarify some aspects of their relationship.

In the polynomial ring $\mathbb{R}[a_{ij}, b_{ij}, c_{ij}, \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$, abbreviated to $\mathbb{R}[\cdot]$ in the following, when considering all constraints generated by B and T, we are to deal with the ideal $\mathbf{I} =< B, T >$ generated by B and T and look for a kind of minimal generating set for the ideal of all constraints.

It can be shown firstly that all constraints of three views are minimally generated by 3 independent constraints in general cases. This can be easily proved as follows: A general 6×4 matrix of full rank has 6×4 d.o.f. (degrees of freedom) and a general 6×4 matrix of rank 3 has only $3 \times 6 + 3$ d.o.f., so the constraints can form only a space of dimension $3 = 4 \times 6 - (3 \times 6 + 3)$.

This suggests that all constraints might be minimally generated by the three bilinearities bb_1, bb_2, bb_3 if they are algebraically independent over $\mathbb{R}(a_{ij}, b_{ij}, c_{ij})$. In the following proposition, we will show that this is almost true except for some *degenerate* cases. Here we stress the distinction between the degenerate cases of points and the degenerate cases of views. This distinction is crucial for the later clarification of the relations among the constraints.

3.1 Degenerate View/point configurations

The degenerate case of points corresponds to the fact that the viewing positions of cameras are in a general configuration, and the points lie in a special situation. For instance, the points in space lie on the trifocal plane. Algebraically this corresponds to work over the polynomial ring $\mathbb{R}(a_{ij}, b_{ij}, c_{ij})[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$ in which we allow denominators depending only on the a_{ij}, b_{ij} and c_{ij}. That is, $\mathbf{u}_i$ are variables and a_{ij}, b_{ij}, c_{ij} are independent *parameters*, or the coefficients of polynomials in $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ are in $\mathbb{R}(a_{ij}, b_{ij}, c_{ij})$ instead of $\mathbb{R}$.

The degenerate case of views corresponds to the fact that the points are general, not lying in any special configurations and the configuration of viewing positions is special. For instance, the three projection centers are collinear. This corresponds to work over the polynomial ring $\mathbb{R}[a_{ij}, b_{ij}, c_{ij}, \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$, so all a_{ij}, b_{ij}, c_{ij} and $\mathbf{u}_i$ are considered as variables.

In the following sections, we will restrict ourselves to $I\!R(a_{ij}, b_{ij}, c_{ij})[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$ for simplicity of computation. Therefore the degenerate cases of views are not considered in this section and will be discussed later.

3.2 Generic points

In this section, without explicit mention, view configurations are considered to be generic.

3.2.1 Bilinearities $\Rightarrow$ trilinearities

Proposition 1 *For any generic point, all trilinear constraints T are contained in the ideal $\mathbf{I}_B$ generated by three algebraically independent bilinear constraints B,* i.e. *all matching constraints of a generic point of three views are contained in the ideal $\mathbf{I}_B$.*

Geometrically, this can be restated as, for the generic points $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ not lying on the tripolar lines, all trilinear constraints of three views are minimally generated by the three epipolar constraints (which are algebraically independent for generic views and points). In other words, the trilinearities are no longer algebraically independent given the bilinear constraints, they are just polynomial consequences of the bilinear ones.

Proof: To prove this proposition, let us generate the trilinearities from the bilineatities by doing some computer algebra. With a computer algebra system, for instance Maple, one can easily check the following results:

$$\begin{aligned}
\mathrm{Res}_{v_2}(bb_1, \mathrm{Res}_{v_3}(bb_2, bb_3)) &= g_1 tt_1, \\
\mathrm{Res}_{u_2}(bb_1, \mathrm{Res}_{v_3}(bb_2, bb_3)) &= g_1 tt_2, \\
\mathrm{Res}_{v_2}(bb_1, \mathrm{Res}_{u_3}(bb_2, bb_3)) &= g_1 tt_3, \\
\mathrm{Res}_{u_2}(bb_1, \mathrm{Res}_{u_3}(bb_2, bb_3)) &= g_1 tt_4,
\end{aligned} \tag{2}$$

where $\mathrm{Res}_x(a, b)$ means the resultant of the polynomials $a(x)$ and $b(x)$ with respect to the variable x. Recall that $\mathrm{Res}_x(a, b)$, eliminates the variable x and generates a polynomial which still belongs to the ideal generated by a and b.

The extra polynomial g_1 appearing in (2) is linear in $\mathbf{u}_1$: $l_1 u_1 + l_2 v_1 + l_3 w_1$ with coefficients in $I\!R[a_{ij}, b_{ij}, c_{ij}]$. This proves

$$g_1 tt_i = \sum f_j bb_j \in \mathbf{I}_B =< bb_1, bb_2, bb_3 > \text{ for } i = 1, \ldots, 4.$$

Continuing in this way, we may obtain the following similar formulae:

$$
\begin{aligned}
\mathrm{Res}_{v_1}(bb_1, \mathrm{Res}_{v_3}(bb_2, bb_3)) &= g_2 tt_5, \\
\mathrm{Res}_{u_1}(bb_1, \mathrm{Res}_{v_3}(bb_2, bb_3)) &= g_2 tt_6, \\
\mathrm{Res}_{v_1}(bb_1, \mathrm{Res}_{u_3}(bb_2, bb_3)) &= g_2 tt_7, \\
\mathrm{Res}_{u_1}(bb_1, \mathrm{Res}_{u_3}(bb_2, bb_3)) &= g_2 tt_8,
\end{aligned} \tag{3}
$$

and

$$
\begin{aligned}
\mathrm{Res}_{v_1}(bb_2, \mathrm{Res}_{v_2}(bb_1, bb_3)) &= g_3 tt_9 \ , \\
\mathrm{Res}_{u_1}(bb_2, \mathrm{Res}_{v_2}(bb_1, bb_3)) &= g_3 tt_{10}, \\
\mathrm{Res}_{v_1}(bb_2, \mathrm{Res}_{u_2}(bb_1, bb_3)) &= g_3 tt_{11}, \\
\mathrm{Res}_{u_1}(bb_2, \mathrm{Res}_{u_2}(bb_1, bb_3)) &= g_3 tt_{12}.
\end{aligned} \tag{4}
$$

As a direct consequence of the above computation, we see that

$$
\begin{aligned}
g_2 tt_i &= \sum f_j bb_j \in \mathbf{I}_B \text{ for } i = 5, \ldots, 8, \\
g_3 tt_i &= \sum f_j bb_j \in \mathbf{I}_B \text{ for } i = 9, \ldots, 12.
\end{aligned}
$$

In summary, we have

$$g_i tt_j \in \mathbf{I}_B \text{ for } i = 1, \ldots, 4 \text{ and } j = 4i-3, \ldots, 4i.$$

It means that the trilinearities are *almost* contained in $\mathbf{I}_B$ except for $g_i = 0$.

Now, the key observation that can be made from the above algebraic development is that the polynomial factors g_i relating the bilinear bb_i and trilinear constraints tt_i are geometrically well defined. The vanishing of each g_i, $g_i = 0$, describes a linear variety which is nothing but the tripolar line in each image! As the epipolar line in the i-th view is given by

$$l_i = (\mathbf{P}_i \mathrm{Ker}(\mathbf{P}_j) \times \mathbf{P}_i \mathrm{Ker}(\mathbf{P}_k))^T \mathbf{u}_i,$$

it is easy to verify that

$$g_i \equiv l_i.$$

The set of degenerate points are therefore completely described by the polynomial

$$g = g_1 g_2 g_3 = 0.$$

For any generic point, *i.e.* $g \neq 0$, it is straightforward that $tt_i \in \mathbf{I}_B$. Thus Proposition 1 is proved.

3.3 Degenerate points

Let examine what happens for degenerate points with generic views.

Proposition 2 *For any degenerate point, each bilinear constraint bb_i is reduced to a linear one g_i and all trilinearities T are reduced to a unique r in reduced form which relates only one coordinate of the same image point across three views. In this case, all constraints are contained in the ideal $\mathbf{I}_R$ generated by the three linearities and the reduced trilinearity $< g_1, g_2, g_3, r >$.*

Geometrically, the reduction of the bilinearities is evident, however, that of the trilinearities needs more symbolic manipulation.

First, it is easy to verify that $\{g_i\}$ is a Grobner basis for $\mathbf{I}_G =< g_1, g_2, g_3 >$. Then:

Lemma 1 *$bb_i = 0$ for $i = 1, 2, 3$ and $g_i = 0 \Rightarrow g_j = 0$ for $j \neq i$.*

Proof: We have immediately $h_1 g_2 \in< bb_1, g_1 >$ and $h_2 g_3 \in< bb_2, g_1 >$, where $h_1 = mv_1 + nw_1$ and $h_2 = m'v_1 + n'w_1$ are generally not vanishing. Geometrically, this lemma means that if the image point lies on the tripolar line in one image, so does its correspondence in the other images.

Now we need to prove the two following lemmas which will complete the proof of the proposition:

Lemma 2 *If $g = 0$, then $bb_i \equiv 0$ modulo $\mathbf{I}_G$ for $i = 1, 2, 3$.*

Proof: It means that in the degenerate case of points, $\mathbf{I}_B \subset \mathbf{I}_G$, the bilinearities are reduced to the linearities. This can be easily checked using any Grobner basis package.

Lemma 3 *If $g = 0$, then $tt_i \equiv r$ modulo $\mathbf{I}_G$ for $i = 1, \ldots 12$, where r is a reduced trilinearity.*

Proof: This lemma states that for the image points lying on the tripolar lines, all trilinearities are equivalent to r (the standard representation) modulo $\mathbf{I}_G$, that is $r \equiv tt_i \mathrm{mod} \mathbf{I}_G$ for $i = 1, \ldots, 12$.

This may be proved by using the normal form reduction with respect to a given basis of an ideal (for instance, the functions available under Maple).

The polynomial r is a reduced trilinearity with 8 terms instead of 12 terms and relates only one non-homogeneous coordinate of the same image point across three views. It is unique for a given term ordering and may have different forms for different term ordering. For example,

$$\begin{aligned} r = \; & \alpha_1 u_1 u_2 u_3 + \alpha_2 u_1 u_2 w_3 + \alpha_3 u_2 u_3 w_1 + \\ & \alpha_4 u_1 u_3 w_2 + \alpha_5 u_1 w_2 w_3 + \alpha_6 u_2 w_1 w_3 + \\ & \alpha_7 u_3 w_1 w_2 + \alpha_8 w_1 w_2 w_3 \end{aligned} \quad (5)$$

for the variables ordered as v_1, v_2, v_3, u_1, u_2, u_3, w_1, w_2, w_3.

3.4 Application to the transfer problem

Among other important applications of these constraints [5, 12, 6], one is that a new (third) image can be predicted provided two images and a number of correspondence points across three views have been established *a priori*.

Using the epipolar geometries to do the transfer [1, 8, 17] corresponds to using the bilinearities bb_1, bb_2, bb_3 to get the trilinearities of a form with 18 terms. Shashua's trilinearities can also be naturally used to do transfer as suggested in [12]. It is known from these authors that the epipolar transfer fails either when the points lie on the trifocal plane or when the projection centers of three views are collinear. In fact, the nature of these two degenerate cases is different, the first one corresponds to degenerate points of generic views and the second the degenerate views of generic points. It is clear that we need the reduced trilinearity to handle the case of degenerate points. For degenerate views, some discussion will be given in Section 5.

4 n views

For general $n \geq 4$ views, there are additional quadrilinearities. It is already known [3, 14] that the quadrilinearities are completely contained in the ideal generated by the bilinearities and the trilinearities. Therefore, we extend the definition of the degenerate points to the general n view case as all those points lying on trifocal planes, defined by any three non-aligned projection centers. Then the following extension to the previous results follows directly as:

Proposition 3 *For any generic point, all matching constraints are contained in the ideal* $\mathbf{I}$ *generated by the* $2n-3$ *algebraically independent bilinear constraints* $< bb_1, bb_2, \ldots, bb_{2n-3} >$*; for any degenerate point, the ideal is generated by the linearities and the reduced trilinearities.*

5 Discussion

This paper clarifies the algebraic relations among the multi-linear matching constraints of n views, especially the relations between the bilinearities and the trilinearities. Though primarily theoritical, it is of practical importance in guiding the uses of the matching constraints.

The exact algebraic relations among the bilinearities, the trilinearities and the quadrilinearities for generic view configurations are established in this paper both for generic and degenerate points. However, the degenerate view configurations are not examined in this paper. It seems [4] that the trilinearities will handle the degenerate view configurations.

The case where the three projection centers are collinear gives an example of degenerate view configuration in which it is obvious that the three bilinearities are algebraically dependent. Theoreitically there is no difficulty in algebraically characterising this situation, as it suffices to consider the polynomial ring $\mathbb{R}[a_{ij}, b_{ij}, c_{ij}, \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$, with relations among the a_{ij}, b_{ij}, c_{ij}, or in other words, the a_{ij}, b_{ij}, c_{ij} not being independent parameters. However in practice this is very memory consuming, and is still under investigation.

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